The Existence of Sterile Neutrino Halos in Galactic Centers as an Explanation of the Black Hole mass - Velocity Dispersion Relation

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ABSTRACT

If sterile neutrinos exist and form halos in galactic centers, they can give rise to observational consequences. In particular, the sterile neutrinos decay radiatively and heat up the gas in the protogalaxy to achieve hydrostatic equilibrium, and they provide the mass to form supermassive blackholes. A natural correlation between the blackhole mass and velocity dispersion thus arises $\log(M_{BH,f}/M_\odot) = \alpha \log(\sigma/200\text{km s}^{-1}) + \beta$ with $\alpha \approx 4$ and $\beta \approx 8$.

Subject headings: Galaxies, Sterile Neutrinos, galactic center, supermassive blackholes, velocity dispersion

1. Introduction

Understanding the nature of dark matter remains a fundamental problem in astrophysics and cosmology. Since the discovery of neutrinos' non-zero rest mass (Fukuda et al. 1998; Bilenky et al. 1998), the possibility that neutrinos contribute to cosmological dark matter has become a hot topic again. In particular, the sterile neutrinos are a class of candidate dark matter particles with no standard model interaction. Although the recent MiniBooNE result challenges the LSND result that suggests the existence of eV scale sterile neutrinos (Aguilar-Arevalo et al. 2007), more massive sterile neutrinos (eg. keV) may still exist. The fact that active neutrinos have rest mass implies that right-handed neutrinos should exist which may indeed be massive sterile neutrinos with rest mass m_s.

Recently, it was proposed that a degenerate sterile neutrino halo exists in the galactic center (Viollier et al. 1993; Munyaneza and Viollier 2002). Later, this model is ruled out with the availability of more precise data, particularly those on orbits of stars such as S2

near the Milky Way center (Schödel et al. 2002). Nevertheless, the existence of such a sterile neutrino halo at the centers of protogalaxies may still be possible. Sterile neutrinos decay into lighter neutrinos and photons, and provide mass to fuel the growth of supermassive blackholes. However, since most of these sterile neutrinos may have either decayed or fallen into the supermassive blackhole, the total mass of the sterile neutrino halo at the galactic center becomes very small ($\ll 10^6 M_\odot$) at present. Moreover, the existence of decaying sterile neutrinos may help to solve the cooling flow problem in clusters (Chan and Chu 2007) as well as reionization in the universe (Hansen and Haiman 2004). Therefore it is worthwhile to discuss the consequences of the existence of massive sterile neutrinos at the centers of protogalaxies, which decay into light neutrinos and photons.

On the other hand, recent observations have led to some tight relations between the central black hole masses $M_{BH,f}$ and velocity dispersions σ in the bulges of galaxies. These relations can be summarized as $\log(M_{BH,f}/M_\odot) = \alpha \log(\sigma/200 \text{ km s}^{-1}) + \beta$, where α is found to be 3.75 ± 0.3 (Gebhardt et al. 2000) and 4.8 ± 0.5 (Ferrarese and Merritt 2000) by two different groups. Tremaine et al. (2002) reanalysed both sets of data and obtained $\alpha = 4.02 \pm 0.32$ and $\beta = 8.13 \pm 0.06$. These results indicate that blackhole formation may be related to galaxy formation, which challenges existing galaxy formation theories (Adams et al. 2001).

This relation has been derived in recent theoretical models (Adams et al. 2001; MacMillan and Henriksen 2002; Robertson et al. 2005; Murray et al. 2005; King 2005, 2003). We assume that a degenerate sterile neutrino halo exists in the center of a protogalaxy. There are two different decay modes for sterile neutrinos ν_s. The major decaying channel is $\nu_s \to 3\nu$ with decay rate (Barger et al. 1995; Boyarsky et al. 2008)

$$\Gamma_{3\nu} = \frac{G_F^2}{384\pi^3} \sin^2 2\theta m_s^5 = 1.77 \times 10^{-20} \sin^2 2\theta \left(\frac{m_s}{1 \text{ keV}}\right)^5 \text{ s}^{-1}, \tag{1}$$

where G_F and θ are the Fermi constant and mixing angle of sterile neutrino with active neutrinos respectively. The minor decaying channel is $\nu_s \to \nu + \gamma$ with decay rate (Barger et al. 1995; Boyarsky et al. 2008)

$$\Gamma = \frac{9\alpha G_F^2}{1024\pi^4} \sin^2 2\theta m_s^5 = 1.38 \times 10^{-22} \sin^2 2\theta \left(\frac{m_s}{1 \text{ keV}}\right)^5 \text{ s}^{-1} \approx \frac{1}{128}\Gamma_{3\nu}, \tag{2}$$

where α is the fine structure constant. It is quite difficult to detect the active neutrinos produced in the major decaying channel. Therefore, we focus on observational consequence of the radiative decay of the sterile neutrinos with rest mass $m_s \geq 10$ keV. They emit high energy photons ($\approx m_s/2$) which heat up the surrounding gas so that hydrostatic equilibrium of the latter is maintained. The sterile neutrino halo also provides mass to form a supermassive blackhole from a small seed blackhole. Without any further assumption, $\alpha \approx 4$ is

consistent with the range of the decay rate obtained by observational data from cooling flow clusters. In this article, we first give a brief review on three popular analytic models that explain the $M_{BH,f} - \sigma$ relation. Then we will give a detailed description of our model and compare it with other existing models.

2. Review on models of the $M_{BH,f} - \sigma$ relation

2.1. Super Eddington Accretion model

King (2003) presented a model to explain the $M_{BH,f} - \sigma$ relation. He assumed that the gas density profile of a protogalaxy is isothermal ($\rho \sim r^{-2}$) (King 2003, 2005). Therefore the gas mass inside radius R is:

$$M(R) = 4\pi \int_0^R \rho r^2 dr = \frac{2 f_g \sigma^2 R}{G}, \tag{3}$$

where $f_g \approx 0.16$ is the cosmological ratio of baryon to total mass, assumed to be the same in a galaxy, and the Virial Theorem is used. Consider a super-Eddington accretion onto a seed blackhole. The accretion feedback produces a momentum-driven superbubble that sweeps ambient gas into a thin shell which expands to the galaxy. The equation of motion is

$$\frac{d}{dt}[M(R)\dot{R}] + \frac{GM(R)[M_{BH}(t) + M(R)]}{R^2} = \frac{L_{edd}}{c}, \tag{4}$$

where $L_{edd} = 4\pi G M_{BH}(t) c/\kappa$, with κ the opacity and $M_{BH}(t)$ the mass of the central blackhole at time t. Integrating twice and assuming $R \gg GM_{BH,f}/\sigma^2$, one gets:

$$R^2 = R_0^2 + 2R_0\dot{R}_0 t - \sigma^2 \left(1 - \frac{M_{BH}(t)}{M_\sigma}\right) t^2, \tag{5}$$

where $\dot{R}_0 = \dot{R}$ at $R = R_0$, with R_0 some large radius ($\gg GM_{BH,f}/\sigma^2$), and $M_\sigma \equiv f_g \kappa \sigma^4/\pi G^2$. Therefore, the maximum radius $R_{\max}$ is given by

$$\frac{R_{\max}^2}{R_0^2} = 1 + \frac{\dot{R}_0^2}{2\sigma^2(1 - M_{BH}(t)/M_\sigma)}. \tag{6}$$

When $M_{BH}(t)$ approaches M_σ, $R_{\max}$ becomes very large such that the cooling of the shocked wind is inefficient as the cooling time $t_{cooling} \propto R^2$ and the accretion is stopped as the shell can escape the galaxy entirely by gas pressure (King 2003). Therefore, given an adequate mass supply (such as in a merger), we get (King 2005)

$$M_{BH,f} = \frac{f_g \kappa}{\pi G^2} \sigma^4. \tag{7}$$

Here, the proportionality constant $f_g \kappa/\pi G^2$ lies within the observational constraints. To summarize, the $M_{BH,f}-\sigma$ relation is obtained with three important assumptions: (1) isothermal gas density distribution throughout the galaxy formation, (2) super Eddington accretion, and (3) an adequate mass supply.

2.2. Self-similar model

MacMillan and Henriksen (2002) obtained a relation between $M_{BH,f}$ and σ by assuming that the density and velocity distributions of matter are self-similar. They assumed that the galaxy is formed by the extended collapse of a halo composed of collisionless matter. The central blackhole is grown proportionally to the halo as matter continues to fall in. The relation is given by (MacMillan and Henriksen 2002)

$$\log M_{BH,f} \propto \left(\frac{3\delta/\alpha - 2}{\delta/\alpha - 1}\right) \log \sigma, \tag{8}$$

where δ and α are scales in space and time respectively, and their ratio is related to the power-law index of the initial density perturbation ϵ in the spherical infall model of halo growth (Henriksen and Widrow 1999):

$$\frac{\delta}{\alpha} = \frac{2}{3}\left(1 + \frac{1}{\epsilon}\right). \tag{9}$$

The power-law index $\epsilon = (n+3)/2$, where n is the index of the primordial matter power spectrum $P(k) \propto k^n$. If $n = -2$, Eq. (8) agrees with the observation $M_{BH,f} \propto \sigma^4$. This model involves a relation Eq. (9) which is quite model dependent.

2.3. Ballistic model

Adams et al. (2001) assume the dark matter and baryons to be unsegregated and the isothermal initial mass density distribution ($M_t(r) \propto r$). The specific orbital energy is conserved when the particles fall into the small seed blackhole:

$$E = \frac{1}{2}v_r^2 + \frac{j^2}{2r^2} - \frac{GM_t(r)}{r}, \tag{10}$$

where v_r and j are the radial velocity and angular momentum per unit mass. When the particles fall into the equatorial plane, their pericenters are

$$p = \frac{j^2}{2GM_t(r)} = \frac{(GM_t(r))^3\Omega^2}{2\sigma^8}, \tag{11}$$

where Ω is the average angular speed of the particles. In the early stage, all the particles fall into the blackhole until the blackhole mass reaches a critical point that corresponds to $p = 4R_s$, where $M_{BH}(t) = M_t(R_s)$, with $R_s = 2GM_{BH}(t)/c^2$ the Schwarzschild radius. This gives the final relation (Adams et al. 2001)

$$M_{BH,f} = \frac{4\sigma^4}{Gc\Omega}. \tag{12}$$

This model is based on the assumption of the isothermal distribution of matter, and there is a free parameter Ω, which is assumed to have the same value for all galaxies.

3. Decaying sterile neutrino halo model

Suppose the sterile neutrino halo dominates the mass in the proto-galactic center, most of the mass in the blackhole comes from the sterile neutrino halo with radius $\tilde{R}$, which was formed in the very early universe $t \sim 0$ (Munyaneza and Biermann 2005; Chan and Chu 2007). The total mass of the degenerate sterile neutrino halo at time t_b is

$$M_s(t_b) = 4\pi \int_0^{\tilde{R}} \rho_s r^2 dr = M_{s0} e^{-\Gamma_{3\nu} t_b}, \tag{13}$$

where ρ_s is the mass density of the sterile neutrino halo. Assume that a seed blackhole with mass $M_{BH,0}$ of order solar mass is formed at t_b, long after the formation of the sterile neutrino halo. It would grow by accreting mass of the sterile neutrino halo to mass $M_{BH}(t)$ at time t. As some sterile neutrinos would be accreted by the seed blackhole, the degenerate pressure is decreased and more sterile neutrinos will fall into the blackhole as their Fermi speed is less than their escape speed (Munyaneza and Biermann 2005). The falling time scale at distance r from the blackhole in a free falling model is given by (Phillips 1994)

$$t_{ff} = \frac{\pi}{2}\sqrt{\frac{r^3}{2G[M_{BH,0} + M_s(r,t)]}}. \tag{14}$$

For $M_s(t) \geq 10^6 M_\odot$, $m_s \geq 10$ keV and $\tilde{R} \leq 0.04$ pc, $t_{ff} \leq 160$ years for $r \leq \tilde{R}$, which is much shorter than the Hubble time. Therefore, we do not need any intermediate mass blackholes since the small seed blackhole can grow to a $10^6 - 10^9 M_\odot$ supermassive blackhole rapidly as long as there is enough mass in the initial sterile neutrino halo. In the following, we assume that all sterile neutrinos fall into the blackhole and decay into active neutrinos and photons, and so $M_{BH,f} = M_s(t_b) + M_{BH,0} \approx M_{s0} e^{-\Gamma_{3\nu} t_b}$.

The photons emitted by the original decaying sterile neutrinos provide energy to the gas in the protogalaxies by Compton scattering. The optical depth of a decayed photon in

the bulge is $\tau = n_e \sigma_T R_e$, where R_e is the J-band effective bulge radius (Marconi and Hunt 2003), σ_T is Compton scattering cross section and n_e is the mean number density of the gas. In equilibrium, the heating rate is equal to the cooling rate by Bremsstrahlung radiation Λ_B, Recombination Λ_R and adiabatic expansion Λ_a. We have (Katz et al. 1996)

$$L(1-e^{-\tau}) = \Lambda_B + \Lambda_R + \Lambda_a = \left[\Lambda_{B0} n_e^2 T^{0.5} + \Lambda_{R0} n_e^2 T^{0.3} \left(1 + \frac{T}{10^6\ \mathrm{K}}\right)^{-1}\right] V + P V^{2/3} c_s, \quad (15)$$

where $\Lambda_{B0} = 1.4 \times 10^{-27}$ erg s^{-1}, $\Lambda_{R0} = 3.5 \times 10^{-26}$ erg s^{-1}, c_s, P and V are the sound speed, pressure and total volume of the gas within R_e respectively. The $M_{BH,f} - \sigma$ relation can be obtained by using Eq. (15) and the Virial theorem numerically. Nevertheless, we first illustrate the idea by obtaining analytic relations in two different regimes. Suppose $M_s(t_b) \geq 10^6 M_\odot$; if $\tau \gg 1$, the resulting temperature is above 10^6 K and the total cooling rate is dominated by Λ_a. For $\tau \leq 1$, the resulting temperature is lower and the total cooling rate is dominated by Λ_B and Λ_R (see Fig. 1).

In the optically thick regime, $\tau \gg 1$ and $\Lambda_a \gg \Lambda_B + \Lambda_R$, and we get

$$kT = \left(\frac{m_g}{\gamma}\right)^{1/3} V^{-4/9} L^{2/3} n_e^{-2/3}, \quad (16)$$

where m_g is the mean mass of a gas particle. By using the Virial theorem $kT = f_1 G M_B m_g / 3R_e$, where M_B is the effective bulge mass of the protogalaxy within R_e, and substituting $L \approx M_s \Gamma c^2/2$, we get

$$M_s = \frac{2\gamma^{1/2} f_1^{3/2} G^{3/2} e^{-\Gamma_{3\nu} t}}{3^{7/6} (4\pi)^{1/3} \Gamma c^2} \left(\frac{M_B}{R_e}\right)^{5/2}, \quad (17)$$

where γ is the adiabatic index of the gas and $f_1 \sim 1$ is a constant that depends on the density distribution of the protogalaxy. As time passes, the energy gained by the gas would decrease gradually and the mass distribution at the center would change slightly also. If the supermassive blackhole was formed when the galaxy formation was nearly completed ($t_b = 10^{16} - 10^{17}$ s), the ratio M_B/R_e and the velocity dispersion do not change significantly. According to Eq. (17), the ratio M_B/R_e is fixed by M_s and Γ. By using the Virial theorem again and assuming spherical symmetry, one can relate this ratio with the final bulge velocity dispersion after the supermassive blackhole is formed,

$$\sigma^2 = f_2 \frac{G M_B}{R_e}, \quad (18)$$

where $f_2 \sim 1$ is a constant that depends on the mass distribution at present. Combining Eq. (17) and Eq. (18), we get

$$M_{BH,f} = M_s(t_b) = \frac{2\gamma^{1/2} f_1^{3/2} e^{-\Gamma_{3\nu} t_b}}{3^{7/6} (4\pi)^{1/3} f_2^{5/2} G \Gamma c^2} \sigma^5. \quad (19)$$

In the optically thin regime, $\tau \leq 1$ and $\Lambda_R + \Lambda_B \gg \Lambda_a$, the total power absorbed by the gas in the protogalaxies within R_e is $\approx L n_e \sigma_T R_e$. If the cooling rate is dominated by Bremsstrahlung radiation, in equilibrium, we get

$$kT = k \left(\frac{L \sigma_T R_e}{\Lambda_0 n_e V} \right)^2 . \tag{20}$$

By using the Virial theorem and Eq. (18), we obtain

$$M_{BH,f} = \frac{2\Lambda_{B0} e^{-\Gamma_{3\nu} t_b}}{\sigma_T f_2^{3/2} G \Gamma c^2} \left(\frac{f_1}{3km_g} \right)^{1/2} \sigma^3 . \tag{21}$$

If the cooling rate is dominated by recombination, in equilibrium and for $T \leq 10^6$ K, we get

$$kT = k \left(\frac{L \sigma_T R_e}{\Lambda_0 n_e V} \right)^{10/3} , \tag{22}$$

and

$$M_{BH,f} = \frac{2\Lambda_{R0} e^{-\Gamma_{3\nu} t_b}}{m_g^{7/10} \sigma_T f_2^{13/10} G \Gamma c^2} \left(\frac{f_1}{3km_g} \right)^{3/10} \sigma^{2.6} . \tag{23}$$

Therefore, $M_{BH,f}$ and σ are closely related in both optically thick and thin regimes.

We use 500 random data in the ranges of $\tau = 0.01 - 10000$, $f_1 = 0.6 - 3$, $f_2 = 0.6 - 3$, $t_b = 10^{16} - 10^{17}$ s, $10^9 M_\odot \leq M_B \leq 10^{12} M_\odot$ and 0.1 kpc $\leq R_e \leq$ 10 kpc to generate the values of $M_{BH,f}$ and σ by using Eq. (15) and (18) (see Fig. 2). By using $\Gamma_{3\nu} = (5 \pm 1) \times 10^{-17}$ s^{-1} which solves the cooling flow problem (Chan and Chu 2007) [1] and $\Gamma_{3\nu} = 128\Gamma$, we get

$$\log \left(\frac{M_{BH,f}}{M_\odot} \right) = (3.98 \pm 0.29) \log \left(\frac{\sigma}{200 \text{ km s}^{-1}} \right) + (8.04 \pm 0.20), \tag{24}$$

which agrees with the recent observation: $\alpha = 4.02 \pm 0.32$ and $\beta = 8.13 \pm 0.06$ (Tremaine et al. 2002).

As an order of magnitude estimate, $\sin\theta \sim m_D/m_s$, where m_D is the neutrino Dirac mass. In the standard see-saw mechanism, m_D is the geometric mean of the light neutrino mass-scale m_ν and m_s (Mohapatra and Senjanović 1980). Therefore, $\sin\theta \sim \sqrt{m_\nu/m_s}$. From Eq. (1), we get

$$\Gamma_{3\nu} \sim 10^{-23} \left(\frac{m_\nu}{1 \text{ eV}} \right) \left(\frac{m_s}{1 \text{ keV}} \right)^4 \text{ s}^{-1} . \tag{25}$$

For $\Gamma_{3\nu} \sim 10^{-17}$ s^{-1}, $m_s \sim 30$ keV for $m_\nu \sim 1$ eV, which is consistent with our assumption ($m_s \geq 10$ keV).

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[1] If we consider the major decaying channel is $\nu_s \to 3\nu$, the decay rate obtained in this paper should correspond to $\Gamma_{3\nu}$.

4. Discussion and summary

We assume the existence of a degenerate neutrino halo ($m_s \sim$ keV) at the proto-galactic center with a parameter, $\Gamma_{3\nu}$, which is universal and can be inferred from observation of cluster hot gas (Chan and Chu 2007). Without any assumptions of the protogalaxy and existence of any intermediate mass blackohles, $\alpha \approx 4$ and $\beta \approx 8$ require the total decay rate to be $\Gamma_{3\nu} = (5 \pm 1) \times 10^{-17}$ s^{-1}, which is consistent with the observational data from cooling flow clusters (Chan and Chu 2007). Also, we have reviewed several models to explain the $M_{BH,f} - \sigma$ relation. These models require several assumptions or free parameters which may not be true for all galaxies. For example, King's model assumes the isothermal density profile ($\rho \sim r^{-2}$) for all galaxies during all the time of the blackhole formation. If the density profile changes into the form $\rho \sim r^{-1}$, then $\sigma \propto \sqrt{r}$ which is not a constant. This problem also exists in the Ballistic model which is based on the isothermal distribution of matter. On the other hand, in the self-similar model, there are several free parameters which are model dependent. Also one cannot obtain the proportionality constant β of the $M_{BH,f} - \sigma$ relation.

We have considered a wide range of $\tau = 0.1 - 10000$, $f_1 = 0.6 - 3$ and $f_2 = 0.6 - 3$ encompassing almost all possibilities in galaxies. In our model, we assume that the supermassive blackhole was formed at the epoch when the galaxy formation was nearly completed ($t_b = 10^{16} - 10^{17}$ s) so that the velocity dispersion does not change significantly between t_b and present. Therefore our result is valid only for supermassive blackholes formed nearly at the end of the galaxy formation, same as in the Super Eddington Accretion and Ballistic model. The assumed existence of a decaying sterile neutrino halo inside each galactic center provides enough mass to form the supermassive blackhole. It can also solve the cooling flow problem in clusters (Chan and Chu 2007) and explain the reionization of the universe (Hansen and Haiman 2004), all with the same decay rate $\Gamma_{3\nu} = (5 \pm 1) \times 10^{-17}$ s^{-1} and $m_s \geq 10$ keV, which are consistent with the standard see-saw mechanism.

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REFERENCES

Adams, F. C. , Graff, D. S. and Richstone, D. O., 2001, ApJL, **551**, L31.

Aguilar-Arevalo, A. A. *et al.* 2007, Phy. Rev. Lett., **98**, 231801.

Barger, V., Phillips, R. J. N. and Sarkar, S. 1995, Phys. Lett. B **352**, 365.

Bilenky, S. M., Giunti, C. and Grimus, W. 1998, European Physical Journal C **1**, 247.

Boyarsky, A., Ruchayskiy, O. and Markevitch, M. 2008, ApJ, **673**, 752.

Chan, M. H. and Chu, M. -C. 2007, ApJ, **658**, 859.

Dodelson, S and Widrow, L. M. 1994, Phys. Rev. Lett., **72**, 17.

Ferrarese, L. and Merritt, D. 2000, ApJL, **539**, L9.

Fukuda, Y. *et al.* 1998, Phys. Rev. Lett., **81**, 1562.

Gebhardt, K., *et al.* 2000, ApJL, **539**, L13.

Hansen, S. H. and Haiman, Z. 2004, ApJ, **600**, 26.

Henriksen, R. N. and Widrow, L. M. 1999, MNRAS, **302**, 321.

Katz, N., Weinberg, D. H. and Hernquist, L. 1996, ApJS, **105**, 19.

King, A. 2003, ApJL, **596**, L27.

King, A. 2005, ApJL, **635**, L121.

MacMillan, J. D. and Henriksen, R. N. 2002, ApJ, **569**, 83.

Marconi, A. and Hunt, L. K. 2003, ApJL, **589**, L21.

McLaughlin, D. E. *et al.* 2006, ApJL, **650**, L37.

Mohapatra, R. N. and Senjanović, G. 1980, Phys. Rev. Lett., **44**, 912.

Munyaneza, F. and Viollier, R. D. 2002, ApJ, **564**, 274.

Munyaneza, F. and Biermann, P. L. 2005, Astronomy and Astrophysics, **436**, 805.

Murray, N., Quataert, E. and Thompson, T. A. 2005, ApJ, **618**, 569.

Phillips, A. C. 1994, *The Physics of Stars*, John Wiley and Sons, Chichester.

Robertson, B. H. *et al.* 2005, ApJ, **641**, 90.

Schödel, R. *et al.* 2002, Nature, **419**, 694.

Tremaine, S. *et al.* 2002, ApJ, **574**, 740.

Viollier, R. D., Trautmann, D. and Tupper, G. B. 1993, Phys. Lett. B., **306**, 79.

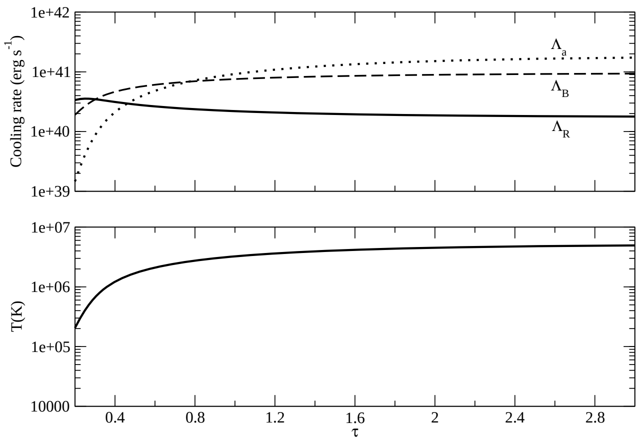

Fig. 1.— Top: The cooling rates by Bremsstrahlung radiation (dashed line), recombination (solid line) and adiabatic expansion (dotted line) versus τ in Eq. (15). Bottom: The temperature of the gas in a protogalaxy versus τ. We used $n_e = 1$ cm^{-3}, $R_e = 1$ kpc and $L = 3 \times 10^{43}$ erg s^{-1}.

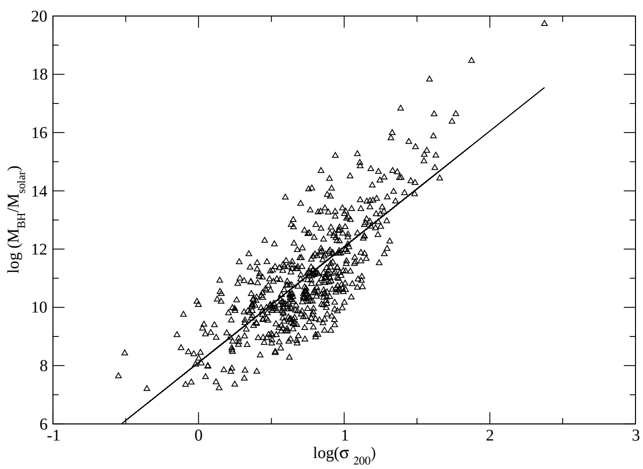

Fig. 2.— $\log M_{BH,f}/M_{\odot}$ versus $\log \sigma_{200}$ for 500 random data, where $\sigma_{200} = \sigma/200\mathrm{km\ s}^{-1}$. We used $\tau = 0.01 - 10000$, $f_1 = 0.6 - 3$, $f_2 = 0.6 - 3$, $t_b = 10^{16} - 10^{17}$ s^{-1}, $M_B = 10^9 - 10^{12} M_{\odot}$ and $R_e = 0.1 - 10$ kpc. The best-fit line in the figure corresponds to slope $\alpha = 3.97 \pm 0.14$ and intercept $\beta = 8.11 \pm 0.12$.